

Name: Solutions
 Class: _____
 Date: _____

ID: A

MAC 2233 Chapter 4 Review for the test

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. Find the derivative of the function.

$$g(x) = 5x^{-3} + 6x^{-6}$$

- a. $g'(x) = 5x^{-3} + 6x^{-6}$
- b. $g'(x) = -15x^{-3} - 36x^{-6}$
- c. $g'(x) = 15x^{-4} + 36x^{-7}$
- d. $g'(x) = -5x^{-4} - 6x^{-7}$
- e. $g'(x) = -15x^{-4} - 36x^{-7}$

$$g'(x) = 5(-3)x^{-3-1} + (6-6)x^{-6-1} = -15x^{-4} - 36x^{-7}$$

2. Find the derivative of the function.

$$r(x) = \frac{2x}{7} - \frac{x^{0.3}}{2} + \frac{4}{7x^{1.3}} - 4$$

- a. $r'(x) = \frac{2}{7} - \frac{0.3}{2x^{0.3}} - \frac{5.2}{7x^{1.3}}$
- b. $r'(x) = \frac{2}{7} - \frac{0.3x^{0.7}}{2} + \frac{5.2x^{2.3}}{7}$
- c. $r'(x) = \frac{2}{7} - \frac{0.3x^{1.3}}{2} - \frac{5.2}{7x^{0.3}}$
- d. $r'(x) = \frac{2}{7} - \frac{0.3}{2x^{0.7}} + \frac{5.2}{7x^{2.3}}$
- e. $r'(x) = \frac{2}{7} - \frac{0.3}{2x^{0.7}} - \frac{5.2}{7x^{2.3}}$

$$r'(x) = \frac{2}{7} - \frac{1}{2}(0.3)x^{0.3-1} + \frac{4}{7}(-1.3)x^{-1.3-1} - 4 = \frac{2}{7} - \frac{0.3}{2}x^{-0.7} - \frac{5.2}{7}x^{-2.3}$$

3. Find the derivative of the function.

$$s(x) = 2\sqrt{x} + \frac{39}{\sqrt{x}} = 2x^{1/2} + 39x^{-1/2}$$

a. $s'(x) = \frac{1}{\sqrt{x}} + \frac{19.5}{x\sqrt{x}}$

b. $s'(x) = \frac{1}{\sqrt{x}} + \frac{19.5}{x\sqrt{x}}$

c. $s'(x) = \frac{2}{\sqrt{x}} - \frac{39}{x\sqrt{x}}$

d. $s'(x) = \frac{1}{\sqrt{x}} - \frac{19.5}{x\sqrt{x}}$

e. $s'(x) = \frac{1}{\sqrt{x}} + \frac{39}{x\sqrt{x}}$

$$s'(x) = 2\left(\frac{1}{2}\right)x^{1/2-1} + 3\left(-\frac{1}{2}\right)x^{-1/2-1}$$

$$= x^{-1/2} - \frac{3}{2}x^{-3/2}$$

$$= \frac{1}{\sqrt{x}} - \frac{3}{2x^{3/2}}$$

$$= \frac{1}{\sqrt{x}} - \frac{39}{2x\sqrt{x}}$$

$$\text{with } \frac{39}{2} = 19.5$$

4. Find the slope of the tangent to the graph of the given function
- $f(x) = 2x^3$
- at the point
- $(-3, -54)$
- .

a. $f'(-3) = 54$

b. $f'(-3) = -162$

c. $f'(-3) = -18$

d. $f'(-3) = 18$

e. $f'(-3) = 0$

$$f'(x) = 2(3)x^{3-1} = 6x^2$$

$$f'(-3) = 6(-3)^2 = 54 = m_{\text{tan}}$$

5. Find the slope of the tangent to the graph of the given function at the indicated point.

$$g(t) = \frac{7}{t^3}, (0.5, 56)$$

a. $g'(0.5) = 112.892$

b. $g'(0.5) = -336$

c. $g'(0.5) = 336$

d. $g'(0.5) = -672$

e. $g'(0.5) = -168$

$$g(t) = 7t^{-3}$$

$$g'(t) = -21t^{-4}$$

$$g'(0.5) = \frac{-21}{(0.5)^4} = -336$$

6. Find all the values of
- x
- (if any) where the tangent line to the graph of the given equation is horizontal.

$$y = 4x^2 + 13x + 13$$

$$(m=0)$$

a. $x = 6.5$

b. $x = 1.63$

c. $x = -6.5$

d. $x = -1.63$

e. $x = 0$

$$y' = 8x + 13 = 0$$

$$8x = -13$$

$$x = \frac{-13}{8} = -1.63$$

7. Find the derivative of the function.

$$h(x) = x(10 + 7x) = 10x + 7x^2$$

$$h'(x) = \boxed{10 + 14x}$$

- a. $17x$
 b. $10 + 14x$
 c. $14 + x$
 d. 7
 e. $10x$

8. Calculate
- $\frac{dy}{dx}$
- . You need not expand your answer.

$$y = (10x^2 + x)(x - x^2)$$

- a. $(20x + 1)(1 - x) + (x - x^2)(10x^2 + x)$
 b. $-40x^2 + 22x + 1$
 c. $(20x + 1)(x - x^2) + (1 - 2x)(10x^2 + x)$
 d. $(20x + 1)(x - x^2) - (1 - 2x)(10x^2 - x)$
 e. $(20x + 1)(1 - x) + (x - 2x^2)(10x^2 + x)$

9. Calculate
- $\frac{dy}{dx}$
- . You need not expand your answer.

$$y = \left(\frac{x}{3.6} + \frac{3.6}{x}\right)(x^2 + 4) = \left(\frac{1}{3.6}x + \frac{3.6}{x}\right) \cdot (x^2 + 4)$$

- a. $2x$
 b. $2x\left(\frac{1}{3.6} - \frac{3.6}{x^2}\right) + \left(\frac{x}{3.6} + \frac{3.6}{x}\right)(x^2 + 4)$
 c. $2x\left(\frac{1}{3.6} - \frac{3.6}{x^2}\right)$
 d. $\left(\frac{1}{3.6} - \frac{3.6}{x^2}\right)(x^2 + 4) + 2x\left(\frac{x}{3.6} + \frac{3.6}{x}\right)$
 e. $\left(\frac{1}{3.6} - \frac{3.6}{x^2}\right)(x^2 + 4) - 2x\left(\frac{x}{3.6} + \frac{3.6}{x}\right)$

product rule

$$f = 10x^2 + x \quad g = x - x^2$$

$$f' = 20x + 1 \quad g' = 1 - 2x$$

$$\text{derivative} = f'g + g'f$$

$$f = \frac{1}{3.6}x + 3.6x^{-1} \quad g = x^2 + 4$$

$$f' = \frac{1}{3.6} - 3.6x^{-2} \quad g' = 2x$$

$$= \frac{1}{3.6} - \frac{3.6}{x^2}$$

10. Calculate $\frac{dy}{dx}$.

$$y = x^2(2x+3)(5x+5)$$

a. $25x^2 + (2x+3)(5x+5)$

b. $40x^3 + 75x^2 + 30x$

c. $65x^2 + (2x+75)(5x+5)$

d. $2x^3 + 75x^2 + 30x$

e. $2x^2 + 75x + 30$

multiply first

$$= x^2(10x^2 + 10x + 15x + 15)$$

$$= x^2(10x^2 + 25x + 15)$$

$$y = 10x^4 + 25x^3 + 15x^2$$

$$y' = 10(4)x^{4-1} + 25(3)x^{3-1} + 15(2)x^{2-1}$$

$$= 40x^3 + 75x^2 + 30x$$

11. Calculate $\frac{dy}{dx}$.

$$y = (\sqrt{x} + 4) \left(\sqrt{x} + \frac{4}{x^2} \right)$$

product rule

a. $\frac{1}{\sqrt{x}} \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\frac{1}{\sqrt{x}} - \frac{8}{x^3} \right) (\sqrt{x} + 4)$

b. $\frac{1}{2\sqrt{x}} \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\frac{1}{2\sqrt{x}} + \frac{8}{x^3} \right) (\sqrt{x} + 4)$

c. $\frac{1}{2\sqrt{x}} \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\frac{1}{2\sqrt{x}} - \frac{8}{x^3} \right) (\sqrt{x} + 4)$

d. $\frac{\sqrt{x}}{2} \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\frac{\sqrt{x}}{2} - \frac{8}{x^3} \right) (\sqrt{x} + 4)$

e. $\frac{1}{2\sqrt{x}} \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\frac{1}{2\sqrt{x}} - 8x \right) (\sqrt{x} + 4)$

$$f = \sqrt{x} + 4$$

$$= x^{1/2} + 4$$

$$f' = \frac{1}{2}x^{-1/2} + 0$$

$$g = \sqrt{x} + \frac{4}{x^2}$$

$$= x^{1/2} + 4x^{-2}$$

$$g' = \frac{1}{2}x^{-1/2} - 8x^{-3}$$

$$f'g + g'f$$

$$= \left(\frac{1}{2}x^{-1/2} \right) \left(x^{1/2} + 4x^{-2} \right) + \left(x^{1/2} + 4 \right) \left(\frac{1}{2}x^{-1/2} - 8x^{-3} \right)$$

$$= \left(\frac{1}{2} \cdot \frac{1}{\sqrt{x}} \right) \left(\sqrt{x} + \frac{4}{x^2} \right) + \left(\sqrt{x} + 4 \right) \left(\frac{1}{2\sqrt{x}} - \frac{8}{x^3} \right)$$

12. Calculate $\frac{dy}{dx}$. You need not expand your answer.

$$y = \frac{5x+5}{4x-1}$$

a. $\frac{5(4x-1) + 4(5x+5)}{(4x-1)^2}$

b. $\frac{5(4x-1) - 4(5x+5)}{(4x-1)^2}$

c. $\frac{5(4x-1) + 4(5x+5)}{4x-1}$

d. $5(4x-1) - 4(5x+5)$

e. 1.25

Quotient Rule

$$f = 5x + 5$$

$$g = 4x - 1$$

$$f' = 5$$

$$g' = 4$$

$$\frac{f'g - g'f}{g^2}$$

$$= \frac{5(4x-1) - 4(5x+5)}{(4x-1)^2}$$

13. Calculate $\frac{dy}{dx}$. You need not expand your answer.

$$y = \frac{2x-3}{(x-5)(x-1)(x-4)}$$

a. $\frac{2(x-5)(x-1)(x-4) + (3x^2 - 20x + 10)(2x-3)}{((x-5)(x-1)(x-4))^2}$

b. $\frac{2}{3x^2 - 20x + 10}$

c. $\frac{2(x-5)(x-1)(x-4) - (3x^2 - 20x + 10)(2x-3)}{(x-5)(x-1)(x-4)}$

d. $\frac{2(x-5)(x-1)(x-4) - (3x^2 - 20x + 29)(2x-3)}{((x-5)(x-1)(x-4))^2}$

e. $\frac{2(x-5)(x-4) - (3x^2 - 20x + 29)}{((x-5)(x-4))^2}$

$$y' = \frac{f'g - g'f}{g^2}$$

$$f = 2x - 3$$

$$g = (x-5)(x-1)(x-4)$$

triple product rule

$$f' = 2$$

$$g' = (1)(x-1)(x-4) + (x-5)(1)(x-4) + (x-5)(x-1)(1)$$

$$y' = \frac{2(x-5)(x-1)(x-4) - (2x-3)[(x-1)(x-4) + (x-5)(x-4) + (x-5)(x-1)]}{((x-5)(x-1)(x-4))^2}$$

14. Compute the derivative.

$$\frac{d}{dx} [(x^3 + 3x)(x^2 - x)] \Big|_{x=2}$$

$$(x^3 + 3x)(x^2 - x) = x^5 - x^4 + 3x^3 - 3x^2$$

- a. 92
b. 36
c. 59
d. 72
e. 78

$$\frac{d}{dx} (x^5 - x^4 + 3x^3 - 3x^2)$$

$$= 5x^4 - 4x^3 + 9x^2 - 6x \Big|_{x=2}$$

15. Calculate the derivative of the function.

$$g(x) = (2x^2 + 2x + 3)^{-3}$$

$$= 5(2)^4 - 4(2)^3 + 9(2)^2 - 6(2) = 72$$

a. $g'(x) = -3(4x+2)(2x^2+2x+3)^{-4}$

b. $g'(x) = (-6x^2+6x+9)^{-4}$

c. $g'(x) = -3(4x+2)(2x^2+2x+3)$

d. $g'(x) = -3(2x^2+2x+3)^{-4}$

e. $g'(x) = -12(2x^2+2x+3)^{-4}$

$$g(x) = (2x^2 + 2x + 3)^{-3}$$

$$g'(x) = -3($$

16. Calculate the derivative of the function.

$$s(x) = \left(\frac{6x+7}{5x-2}\right)^5 = (6x+7)^5 (5x-2)^{-5}$$

a. $s'(x) = \left(\frac{6x+7}{5x-2}\right)^4 \frac{47}{(5x-2)^2}$

b. $s'(x) = -5 \left(\frac{6x+7}{5x-2}\right)^4 \frac{12}{(5x-2)^2}$

c. $s'(x) = -5 \left(\frac{6x+7}{5x-2}\right)^4 \frac{47x}{(5x-2)^2}$

d. $s'(x) = 5 \left(\frac{6x+7}{5x-2}\right)^4$

e. $s'(x) = -5 \left(\frac{6x+7}{5x-2}\right)^4 \frac{47}{(5x-2)^2}$

$$f = (6x+7)^5$$

$$f' = 5(6x+7)^4 (6)$$

$$= 30(6x+7)^4$$

$$g = (5x-2)^{-5}$$

$$g' = -5(5x-2)^{-6} \cdot 5$$

$$= -25(5x-2)^{-6}$$

$$s'(x) = 30(6x+7)^4 (5x-2)^{-5} + -25(5x-2)^{-6} (6x+7)^5$$

$$= -5(6x+7)^4 (5x-2)^{-6} [-6(5x-2) + 5(6x+7)]$$

$$= \frac{-5(6x+7)^4 \cdot 47}{(5x-2)^6}$$

17. Find the indicated derivative. The independent variable is a function of t .

$$y = x^{0.5}(1+x); \frac{dy}{dt} = ? \quad = x^{0.5} + x^{1.5}$$

a. $\frac{dy}{dt} = (1.5x^{0.5}) \frac{dx}{dt}$

b. $\frac{dy}{dt} = (0.5x^{-0.5}) \frac{dx}{dt}$

c. $\frac{dy}{dt} = (0.5x^{-0.5} + 2.5x^{0.5}) \frac{dx}{dt}$

d. $\frac{dy}{dt} = (0.5x^{-0.5} + 1.5x^{0.5}) \frac{dx}{dt}$

e. $\frac{dy}{dt} = (0.5x^{0.5} + 2.5x^{0.5}) \frac{dx}{dt}$

$$\frac{dy}{dt} = 0.5x^{-0.5} \frac{dx}{dt} + 1.5x^{0.5} \frac{dx}{dt}$$

$$= [0.5x^{-0.5} + 1.5x^{0.5}] \frac{dx}{dt}$$

This is a type of implicit differentiation

18. Find the indicated derivative.

$$y = 8x^3 + \frac{11}{x}, x = 5 \text{ when } t = 1, \left. \frac{dx}{dt} \right|_{t=1} = 11; \left. \frac{dy}{dt} \right|_{t=1} = ?$$

Please round the answer to the nearest hundredth.

a. $\left. \frac{dy}{dt} \right|_{t=1} = 2175.80$

b. $\left. \frac{dy}{dt} \right|_{t=1} = 599.56$

c. $\left. \frac{dy}{dt} \right|_{t=1} = 1315.16$

d. $\left. \frac{dy}{dt} \right|_{t=1} = 6595.16$

e. $\left. \frac{dy}{dt} \right|_{t=1} = 13151.60$

$$y = 8x^3 + 11x^{-1}$$

$$\frac{dy}{dt} = 24x^2 \frac{dx}{dt} + -11x^{-2} \frac{dx}{dt}$$

$$\frac{dy}{dt} = 24(5)^2(11) + -11(5)^{-2}(11)$$

$$\frac{dy}{dt} = 6595.16$$

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19. Find the derivative of the following function.

$$f(x) = \ln(5x - 9)$$

a. $\frac{1}{5x-9}$

b. $\frac{5}{5x-9}$

c. $\frac{9}{5x-9}$

d. $\frac{45}{5x-9}$

e. none of these

$$f(x) = \ln(5x-9)$$

$$f'(x) = \frac{1}{5x-9} \cdot \frac{d}{dx}(5x-9)$$

$$= \frac{5}{5x-9}$$

5

20. Find the derivative of the following function.

$$f(x) = \log_7 4x$$

a. $\frac{1}{4x \ln(7)}$

b. $\frac{7}{x \ln(4)}$

c. $\frac{1}{x \ln(7)}$

d. $\frac{4}{x \ln(7)}$

e. none of these

$$f(x) = \log_a x$$

$$f'(x) = \frac{1}{x \ln a}$$

$$\log_7 4x = f(x)$$

$$f'(x) = \frac{1}{4x \cdot \ln 7} \cdot 4$$

$$= \frac{1}{x \ln 7}$$

chain

21. Find the derivative of the function.

$$f(x) = (x^9 + 8) \ln x$$

a. $\frac{x^9(9+9\ln x) + 8}{x}$

b. $\frac{x^8(1+9\ln x) + 8}{x}$

c. $\frac{x^9(1+9\ln x) + 8}{x}$

d. $\frac{x^9(1+\ln x) + 8}{x}$

e. none of these

product

$$f = x^9 + 8$$

$$f' = 9x^8$$

$$g = \ln x$$

$$g' = \frac{1}{x}$$

$$= 9x^8(\ln x) + \frac{1}{x}(x^9 + 8)$$

$$= 9x^8 \cdot \ln x + x^8 + \frac{8}{x}$$

$$= \frac{9x^9 \ln x}{x} + \frac{x^9}{x} + \frac{8}{x}$$

$$= \frac{x^9(\ln x + 1) + 8}{x}$$

22. Find the derivative of the function.

$$h(x) = \ln[(-2x+2)(7x+5)] = \ln(-2x+2) + \ln(7x+5)$$

a. $\frac{7}{(-2x+2)} + \frac{2}{(7x+5)}$

b. $\frac{7}{(-2x+2)} - \frac{2}{(7x+5)}$

c. $\frac{-2}{(-2x+2)} + \frac{7}{(7x+5)}$

d. $\frac{1}{(-2x+2)} + \frac{1}{(7x+5)}$

e. $\frac{1}{(-2x+2)} - \frac{1}{(7x+5)}$

$$h'(x) = \frac{1}{-2x+2} \cdot -2 + \frac{1}{7x+5} \cdot 7$$

$$= \frac{-2}{-2x+2} + \frac{7}{7x+5}$$

23. Find the derivative of the function.

$$f(x) = \ln \left| \frac{(5x+3)^6}{(4x+2)^9(8x+9)} \right| = \ln(5x+3)^6 - \ln(4x+2)^9 - \ln(8x+9)$$

a. $\frac{30}{5x+3} + \frac{36}{4x+2} + \frac{8}{8x+9}$

b. $\frac{30}{5x+3} - \frac{36}{4x+2} - \frac{8}{8x+9}$

c. $\frac{5}{(5x+3)^6} - \frac{4}{(4x+2)^9} - \frac{8}{8x+9}$

d. $\frac{5}{(5x+3)^6} + \frac{4}{(4x+2)^9} + \frac{8}{8x+9}$

e. none of these

$$= 6 \ln(5x+3) - 9 \ln(4x+2) - \ln(8x+9)$$

$$f'(x) = 6 \cdot \frac{1}{5x+3} \cdot 5 - 9 \cdot \frac{1}{4x+2} \cdot 4 - \frac{1}{8x+9} \cdot 8$$

$$= \frac{30}{5x+3} - \frac{36}{4x+2} - \frac{8}{8x+9}$$

24. Find the derivative of the function.

$$r(x) = [\ln(x^7)]^4 = [7 \cdot \ln x]^4$$

a. $\frac{28[\ln(x^6)]^3}{x^7}$

b. $\frac{28[\ln(x^7)]^3}{x^7}$

c. $\frac{28[\ln(x^7)]^3}{x}$

d. $\frac{28[\ln(x^7)]^4}{x^7}$

e. none of these

$$r'(x) = 4 \cdot [7 \ln x]^3 \cdot \frac{d}{dx}(7 \ln x)$$

$$= 4 [7 \ln x]^3 \cdot \frac{7}{x}$$

rewrite
as 7
 $\ln x$
again

$$= \frac{28 [\ln x^7]^3}{x}$$

25. Find the derivative of the function.

$$f(x) = e^{5x^7} \ln 4x$$

Product
Rule

a. $35e^{5x^7} x^6 \ln 4x + \frac{e^{5x^7}}{x}$

b. $35e^{5x^7} x^6 \ln 4x + \frac{e^{5x^7}}{4}$

c. $35e^{5x^6} x^6 \ln 4x + \frac{e^{5x^7}}{x}$

d. $35e^{5x^7} x^7 \ln 4x + \frac{4e^{5x^7}}{x}$

e. $7e^{5x^7} x^6 \ln 4x + \frac{4e^{5x^7}}{x}$

$$f = e^{5x^7} \quad g = \ln 4x$$

$$f' = e^{5x^7} \cdot 35x^6$$

$$g' = \frac{1}{4x} \cdot 4 = \frac{1}{x}$$

$$= 35x^6 \cdot e^{5x^7} \cdot \ln 4x$$

$$+ \frac{1}{x} \cdot e^{5x^7}$$

$$= 35e^{5x^7} \cdot x^6 \ln 4x + \frac{e^{5x^7}}{x}$$

26. Find the derivative of the function.

$$h(x) = e^{5x^2 - 2x + \frac{1}{x}}$$

a. $\frac{10x^2 - 2x - 1}{x} e^{5x^2 - 2x + \frac{1}{x}}$

b. $\frac{10x^3 - 2x^2 - 1}{x^2} e^{5x^2 - 2x + \frac{1}{x}}$

c. $\frac{5x^3 - 4x^2 - 1}{x} e^{5x^2 - 2x + \frac{1}{x}}$

d. $\frac{5x^3 - 4x^2 - 1}{x^2} e^{5x^2 - 2x + \frac{1}{x}}$

e. none of these

$$h'(x) = e^{5x^2 - 2x + \frac{1}{x}} \cdot \frac{d}{dx} \left(5x^2 - 2x + \frac{1}{x} \right)$$

$$h'(x) = e^{5x^2 - 2x + \frac{1}{x}} \cdot (10x - 2 - x^{-2})$$

lcm = x^2

$$= \frac{10x^3 - 2x^2 - 1}{x^2} e^{5x^2 - 2x + \frac{1}{x}}$$

27. Find the derivative of the function.

$$\frac{e^{-10x}}{10xe^{10x}} = \text{Algebra 1 first} = \frac{1}{10x e^{20x}} = \frac{e^{-20x}}{10x}$$

Quotient Rule

a. $-\frac{20x - 1}{10x^2 e^{20x}}$

b. $-\frac{20x + 1}{x^2 e^{20x}}$

c. $\frac{20x + 1}{10x^2 e^{20x}}$

d. $-\frac{20x + 1}{10x^2 e^{20x}}$

e. none of these

$f = e^{-20x}$ $g = 10x$

$f' = -20e^{-20x}$ $g' = 10$

$$= \frac{-200x e^{-20x} + 10 e^{-20x}}{(10x)^2}$$

28. Find $\frac{dy}{dx}$ using implicit differentiation.

$$3x + 4y = 10$$

a. $-\frac{4}{3}$

b. $-\frac{3}{4}$

c. $-\frac{3}{4}$

d. 0

e. -4

$$3 + 4y' = 0$$

$$4y' = -3$$

$$y' = -\frac{3}{4}$$

$$= \frac{-40e^{-20x} [20x + 1]}{1000x^2}$$

$$= -\frac{20x + 1}{10x^2 e^{20x}}$$

29. Find $\frac{dy}{dx}$ using implicit differentiation.

$$7x + 5y = xy$$

a. $\frac{x-5}{7-y}$

b. $\frac{7-y}{x-5}$

c. $5-y$

d. $x-7$

e. $\frac{7-x}{y-5}$

$$7 + 5y' = y + xy'$$

$$5y' - xy' = y - 7$$

$$y'(5-x) = y-7$$

$$y' = \frac{y-7}{5-x}$$

$$= \frac{7-y}{x-5}$$

$$7x + 5y = \underbrace{xy}_{\text{product}}$$

$$f=x \quad g=y$$

$$f'=1 \quad g'=y'$$

$$y + xy'$$

30. Find $\frac{dy}{dx}$ using implicit differentiation.

$$y \ln x + y = 10$$

a. $-\frac{y}{x(\ln x + 1)}$

b. $-\frac{x}{y(\ln y + 1)}$

c. $\frac{y}{x \ln x}$

d. $-\frac{1}{x(\ln x + 1)}$

e. $\frac{y}{x(\ln x + 1)}$

$$\underbrace{y \ln x + y}_{\text{product}} = 10$$

$$f=y \quad g=\ln x$$

$$f'=y' \quad g'=\frac{1}{x}$$

$$y' \ln x + \frac{y}{x}$$

$$y' \ln x + \frac{y}{x} + y' = 0$$

$$y' \ln x + y' = -\frac{y}{x}$$

$$y'(\ln x + 1) = -\frac{y}{x}$$

$$y' = \frac{-y}{x(\ln x + 1)}$$

31. Find $\frac{dy}{dx}$ using implicit differentiation.

$$\frac{xy}{8} - y^2 = 5$$

a. $\frac{1}{\sqrt{16xy}}$

b. $\frac{y}{16x - y}$

c. $\frac{16y - 8x}{8y - x}$

d. $\frac{1}{8y - x}$

e. $\frac{y}{16y - x}$

$$\frac{1}{8}xy - y^2 = 5$$

product

$$f = \frac{1}{8}x \quad g = y$$

$$f' = \frac{1}{8} \quad g' = y'$$

$$\frac{1}{8}y + \frac{1}{8}xy' - 2y \cdot y' = 0$$

$$\frac{1}{8}xy' - 2y \cdot y' = -\frac{1}{8}y$$

multiply by 8

$$xy' - 16y \cdot y' = -y$$

$$y'(x - 16y) = -y$$

$$y' = \frac{-y}{x - 16y} = \frac{y}{16y - x}$$

32. Find $\frac{dx}{dy}$ using implicit differentiation.

$$(xy)^2 + y^2 = 3$$

a. $2y + 2x$

b. $-\frac{(x^2 + 1)}{xy}$

c. $-\frac{x}{y}$

d. $\frac{xy}{x^2 + 1}$

e. $\frac{xy}{(x^2 + 1)}$

$$\left. \begin{aligned} (xy)^2 + y^2 &= 3 \\ x^2 y^2 + y^2 &= 3 \\ y^2(x^2 + 1) &= 3 \end{aligned} \right\} \text{algebra}$$

$$f = y^2 \quad g = x^2 + 1$$

product rule

$$f' = 2y \cdot y' \quad g' = 2x$$

Answer is not among the choices!

$$(2y \cdot y' (x^2 + 1) + 2xy^2) = 0$$

$$2yy'(x^2 + 1) = -2xy^2$$

$$y' = \frac{-2xy^2}{2y(x^2 + 1)} = \frac{-xy}{x^2 + 1}$$

$$\frac{-xy}{x^2 + 1}$$

33. Find $\frac{dy}{dx}$ using implicit differentiation.

$$xe^y - ye^x = 10$$

a. $\frac{y-1}{x-1}$

b. $\frac{xe^x + e^y}{ye^y + e^x}$

c. $\frac{xe^y - e^x}{ye^x - e^y}$

d. $\frac{ye^x - e^y}{xe^y - e^x}$

e. $\frac{ye^y + e^x}{xe^y - e^x}$

xe^y

$$f = x \quad g = e^y$$

$$f' = 1 \quad g' = e^y \cdot y'$$

$$e^y + x \cdot e^y \cdot y'$$

ye^x

$$f = y \quad g = e^x$$

$$f' = y' \quad g' = e^x$$

$$y' e^x + e^x \cdot y$$

34. Find $\frac{dy}{dx}$ using implicit differentiation.

$$\frac{e^x}{y^2} = 12 + e^y$$

a. $\frac{y}{2+y^3}$

b. $\frac{ye^x}{2e^x + y^3 e^y}$

c. $\frac{2e^x + y^3 e^y}{ye^x}$

d. $\frac{ye^x}{12e^x + 3y^2 e^y}$

e. $\frac{2ye^x}{e^x + ye^y}$

$$e^y + x e^y \cdot y' - y' e^x - e^x y = 0$$

$$x e^y \cdot y' - y' e^x = e^x y - e^y$$

$$y' (x e^y - e^x) = e^x y - e^y$$

$$y' = \frac{y \cdot e^x - e^y}{x \cdot e^y - e^x}$$

$e^x \cdot y^{-2}$

$$f = e^x \quad g = y^{-2}$$

$$f' = e^x \quad g' = -2y^{-3} \cdot y'$$

$$e^x \cdot y^{-2} + e^x (-2y^{-3}) y'$$

$$e^x y^{-2} + e^x (-2y^{-3}) y' = 0 + e^y \cdot y'$$

$$e^x (-2y^{-3}) y' - e^y \cdot y' = -e^x y^{-2}$$

$$y' \left(-\frac{2e^x}{y^3} - e^y \right) = -\frac{e^x}{y^2}$$

multiply by (y^3)

$$y' = \frac{e^x y}{2e^x + y^3 e^y}$$

35. Find $\frac{dy}{dx}$ using implicit differentiation.

$$\ln(20 + e^{xy}) = y$$

a. $x + y$

b. $\frac{1}{20 + e^{xy}(1-x)}$

c. $\frac{ye^{xy}}{20 + e^{xy}}$

d. $\frac{y}{1-x}$

e. $\frac{ye^{xy}}{20 + e^{xy}(1-x)}$

$$\frac{1}{20 + e^{xy}} \cdot \frac{d}{dx} (20 + e^{xy}) = y'$$

$f = x \quad g = y$
 $f' = 1 \quad g' = y'$
 $y + xy'$

$$\frac{1}{20 + e^{xy}} \cdot e^{xy} \cdot (y + xy') = y'$$

$$e^{xy}(y + xy') = y'(20 + e^{xy})$$

$$ye^{xy} + xy'e^{xy} = y'(20) + y'e^{xy}$$

36. Use the shortcut rules to calculate the derivative of the given function.

$$f(x) = 8x^{2.5}$$

a. $f(x) = 20x$

b. $f(x) = 8x^{2.5}$

c. $f(x) = 20x^{1.5}$

d. $f(x) = 20x^{2.5}$

e. $f(x) = 8x^{1.5}$

$$xy'e^{xy} - y'e^{xy} - y'(20) = -ye^{xy}$$

$$y'(xe^{xy} - e^{xy} - 20) = -ye^{xy}$$

$$y' = \frac{-ye^{xy}}{xe^{xy} - e^{xy} - 20}$$

multiply by $\frac{-1}{-1}$

$$y' = \frac{ye^{xy}}{e^{xy}(x-1) - 20}$$

Short Answer

37. Find the derivative of the function.

$$s(x) = 7\sqrt{x} + \frac{35}{\sqrt{x}}$$

38. Find the derivative of the function.

$$k(x) = \frac{6x^8 - 10x^9}{x^3}$$

$$y' = \frac{ye^{xy}}{-xe^{xy} + e^{xy} + 20}$$

$$y' = \frac{ye^{xy}}{20 + e^{xy}(1-x)}$$

36

$$f(x) = 8x^{2.5}$$

$$f'(x) = 8(2.5)x^{2.5-1}$$

$$= 20x^{1.5}$$

39. Given.

$$\lim_{x \rightarrow 8} \frac{x^2 - 16x + 64}{x^2 - 8x}$$

$$= \lim_{x \rightarrow 8} \frac{x^2 - 16x + 64}{x^2 - 8x} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 8} \frac{2x - 16}{2x} = \frac{0}{16} = 0$$

$\frac{2(8) - 16}{2(8)}$ ↗

Say whether L'Hospital's rule applies.

It is does, use it to evaluate the given limit. If not, use some other method.

40. Given.

$$\lim_{x \rightarrow -2} \frac{x^2 + 14x + 24}{x^2 + 2x}$$

$$= \lim_{x \rightarrow -2} \frac{x^2 + 14x + 24}{x^2 + 2x} = \frac{0}{0}$$

$$= \lim_{x \rightarrow -2} \frac{2x + 14}{2x + 2}$$

$$= \frac{2(-2) + 14}{2(-2) + 2}$$

Say whether L'Hospital's rule applies.

It is does, use it to evaluate the given limit. If not, use some other method.

$$= \frac{10}{-2} = -5$$

37

$$S(x) = 7\sqrt{x} + \frac{35}{\sqrt{x}} = 7x^{1/2} + 35x^{-1/2}$$

$$S'(x) = 7\left(\frac{1}{2}\right)x^{-1/2} + 35\left(-\frac{1}{2}\right)x^{-3/2}$$

$$= \frac{7 \cdot \frac{1}{2}}{2\sqrt{x}} - \frac{35 \cdot \frac{1}{2}}{2x^{3/2}}$$

38

$$K(x) = \frac{6x^8 - 10x^9}{x^3} = \frac{6x^8}{x^3} - \frac{10x^9}{x^3} = 6x^5 - 10x^6$$

$$K'(x) = 30x^4 - 60x^5$$

**MAC 2233 Chapter 4 Review for the test
Answer Section****MULTIPLE CHOICE**

- | | |
|------------|--------|
| 1. ANS: E | PTS: 1 |
| 2. ANS: E | PTS: 1 |
| 3. ANS: D | PTS: 1 |
| 4. ANS: A | PTS: 1 |
| 5. ANS: B | PTS: 1 |
| 6. ANS: D | PTS: 1 |
| 7. ANS: B | PTS: 1 |
| 8. ANS: C | PTS: 1 |
| 9. ANS: D | PTS: 1 |
| 10. ANS: B | PTS: 1 |
| 11. ANS: C | PTS: 1 |
| 12. ANS: B | PTS: 1 |
| 13. ANS: D | PTS: 1 |
| 14. ANS: D | PTS: 1 |
| 15. ANS: A | PTS: 1 |
| 16. ANS: E | PTS: 1 |
| 17. ANS: D | PTS: 1 |
| 18. ANS: D | PTS: 1 |
| 19. ANS: B | PTS: 1 |
| 20. ANS: C | PTS: 1 |
| 21. ANS: C | PTS: 1 |
| 22. ANS: C | PTS: 1 |
| 23. ANS: B | PTS: 1 |
| 24. ANS: C | PTS: 1 |
| 25. ANS: A | PTS: 1 |
| 26. ANS: B | PTS: 1 |
| 27. ANS: D | PTS: 1 |
| 28. ANS: C | PTS: 1 |
| 29. ANS: B | PTS: 1 |
| 30. ANS: A | PTS: 1 |
| 31. ANS: E | PTS: 1 |
| 32. ANS: B | PTS: 1 |
| 33. ANS: D | PTS: 1 |
| 34. ANS: B | PTS: 1 |
| 35. ANS: E | PTS: 1 |
| 36. ANS: C | PTS: 1 |

SHORT ANSWER

37. ANS:

$$\frac{3.5}{\sqrt{x}} - \frac{17.5}{x^{1.5}}$$

PTS: 1

38. ANS:

$$30x^4 - 60x^5$$

PTS: 1

39. ANS:

yes; 0

PTS: 1

40. ANS:

yes; -5

PTS: 1